Turn in HW: Period 3

Write or find one problem that is to evaluate an improper integral.

Show clear process to support the answer.

Turn in HW: Period 4

Write or find one problem that is to evaluate an integral requiring partial fractions.

Show clear process to support the answer.

Individual Quiz # 1: Friday

By hand

- sketch a polar curve
- ☆ find the equation of a tangent line to a polar curve
- ✓ evaluate an improper integral
- ✓ evaluate an integral requiring partial fractions

With grapher

- graph a polar curve
- find the smallest interval of completion
- classify it and state symmetry

Individual Quiz # 2, possible topics

- Euler's Method
- Integration by parts
- Separation of variables to find a particular solution to a differential equation.

Length of a curve. (Arc length)

Probable date: Tuesday, June 5

Review for Quiz:

Section 9.5: Partial Fractions

Section 9.7: Improper Integrals

9.5 Partial Fractions

for $\int \frac{f(x)}{g(x)}\, dx$

When degree of top < degree of the bottom
and the bottom is factorable

When top > bottom, do the division!

$$\int \frac{x+7}{x^2-x-6}\,dx$$

Rewrite:

$$\frac{x+7}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

Solve Basic Equation for A & B: $x+7 = A(x+2) + B(x-3)$

let $x=-2 \rightarrow -2+7 = A(-2+2) + B(-2-3)$

$5 = 0 + B(-5) \rightarrow B=-1$

$x = 3 \rightarrow 10 = 5A$

$A = 2$

Integrate:

$$\int \frac{x+7}{x^2-x-6}\,dx = \int \frac{2}{x-3}\,dx - \int \frac{1}{x+2}\,dx$$

$$= 2\ln|x-3| - \ln|x+2| + C$$

$$= \ln\left|\frac{(x-3)^2}{x+2}\right| + C$$

Denominators with repeat factors:

$$\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx$$

Rewrite:

$$\frac{5x^2+20x+6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

Solve Basic Equation for A, B & C:

$5x^2+20x+6 = A(x+1)^2 + Bx(x+1) + Cx$

let $x=0$ $\quad 6 = A$

$x=-1$ $\quad 5-20+6 = C(-1) \rightarrow C=9$

$x=1$ $\quad 31 = 6(4) + B(1)(2) + 9(1)$

$B = -1$

Denominators with repeat factors:

Integrate:

$$\int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x}\,dx = \int \left(\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}\right)dx$$

$$= \int \left(\frac{6}{x} + \frac{-1}{x+1} + \frac{9}{(x+1)^2}\right)dx$$

$$= 6\ln x - \ln|x+1| - \frac{9}{x+1} + C$$

Denominators with repeat factors:

$$\int \frac{5}{x^3(x+2)}\,dx = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x+2}$$

Each power of the repeat factor needs its own partial fraction.

Quadratic factors:

$$\int \frac{3x^2+4x+4}{x^3+4x}\,dx$$

Rewrite:

$$\frac{3x^2+4x+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

Solve Basic Equation:

$$3x^2+4x+4 = A(x^2+4) + x(Bx+c)$$

Let $x=0$ $\quad 4 = 4A \rightarrow A=1$

$x=1$ $\quad 11 = 1(5) + 1(B+C)$

$$\boxed{6 = B+C}$$

$x=-1$

$$3-4+4 = 1(5) + (-1)(B(-1)+c)$$

$$\boxed{-2 = B-C}$$

Solve System
$B=2$
$C=4$

Integrate:

$$\int \frac{3x^2+4x+4}{x^3+4x}\,dx = \int \frac{1}{x}\,dx + \int \frac{2x+4}{x^2+4}\,dx$$

$$= \ln|x| + \int \frac{2x}{x^2+4}\,dx + \int \frac{4}{x^2+4}\,dx$$

$$= \ln|x| + \ln(x^2+4) + 2\arctan\left(\frac{x}{2}\right) + C$$

$$= \ln|x^3+4x| + 2\arctan\left(\frac{x}{2}\right) + C$$

9.7 Improper Integrals

Remember: Definition of a Definite Integral

$\int_a^b f(x)\,dx$ f(x) must be continuous on [a, b]

Improper Integrals

If a &/or b = $\pm\infty$

Or if

f(x) has any infinite discontinuities on [a, b]

Infinite Discontinuities when:

$\lim\limits_{x \to c} f(x) = \pm\infty$ from the right or left

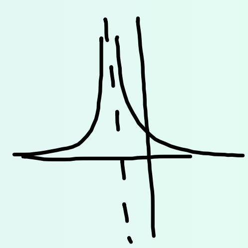
Examples

$\int_{-1}^{\infty} \frac{1}{x}\,dx$; $\int_1^5 \frac{1}{\sqrt{x-1}}\,dx$; $\int_{-2}^{2} \frac{1}{(x+1)^2}\,dx$

undef @ $x=0$
(∞ discontinuity)
AND upper limit ∞

$\lim\limits_{x \to 1^+} = \frac{1}{0}$
$= \infty$

$\lim\limits_{x \to -1} = \infty$



Evaluating Improper Integrals

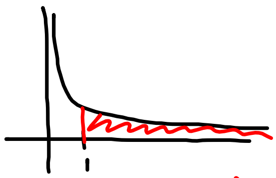
Use a limit process:

$$\int_1^\infty \frac{1}{x^2}\,dx \qquad \lim_{b\to\infty}\int_1^b \frac{1}{x^2}\,dx = \lim_{b\to\infty}\left[-\frac{1}{x}\right]_1^b$$

$$= \lim_{b\to\infty}\left[-\frac{1}{b}+1\right]$$

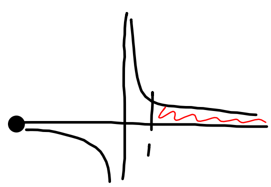
$$= -\cancelto{0}{\frac{1}{\infty}} + 1$$

$$= \boxed{1}$$



The integral
= area under the curve
is converging to $\boxed{1}$

$$\int_1^\infty \frac{1}{x}\,dx$$



$$= \lim_{b\to\infty}\int_1^b \frac{1}{x}\,dx$$

$$= \lim_{b\to\infty}\Big[\ln|x|\Big]_1^b = \lim_{b\to\infty}\left[\ln b - \underset{0}{\ln 1}\right]$$

$$= \ln\infty$$

$$= \infty \quad \text{DNE}$$

* The area is infinite. The integral does not converge. Diverges.

$$\int_{-1}^{2} \frac{1}{x^3}\,dx$$

Infinite discontinuity at x=0. Split the integral and use the limit process to approach the discontinuity from both sides.

$$= \lim_{b \to 0^-} \int_{-1}^{b} \frac{1}{x^3}\,dx + \lim_{a \to 0^+} \int_{a}^{2} \frac{1}{x^3}\,dx$$

$$= \lim_{b \to 0^-} \left[-\frac{1}{2x^2} \right]_{-1}^{b} + \lim_{a \to 0^+} \left[-\frac{1}{2x^2} \right]_{a}^{2}$$

$$= \lim_{b \to 0^-} \left[-\frac{1}{2b^2} + \frac{1}{2} \right] + \lim_{a \to 0^+} \left[-\frac{1}{8} + \infty \right]$$

$$\left(-\infty + \tfrac{1}{2}\right) + \left(-\tfrac{1}{8} + \infty\right)$$

★ Consider each part separately. If either of the improper integrals diverges, the the original integral diverges.

HW: Review for Quiz

Section 9.5: Partial Fractions

Do p. 536, # 1, 9, 17, 31

Section 9.7: Improper Integrals

Do p. 553, # 1 - 11 odd